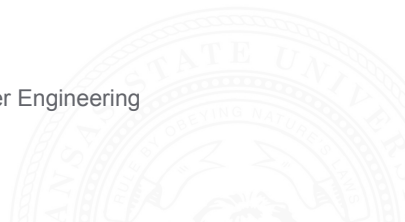


Power Flow I

System quantities and physical equations

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Lecture outline

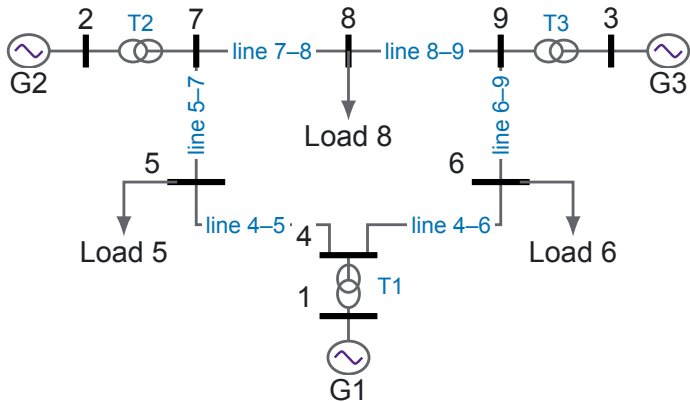
- 1 System components and their voltage, current, and power quantities.
- 2 Physical equations: branch currents, complex power, and bus balance.
- 3 Exercise: write the physical equations for a given network.

PART 1

System components and quantities

Identify the devices and connections in a power system, then name the voltage, current, and power quantities associated with them.

Three-machine, nine-bus single-line diagram



Trace a connected path

G2 → bus 2

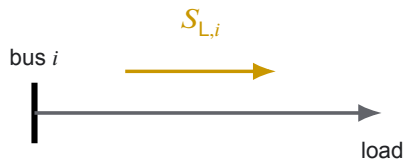
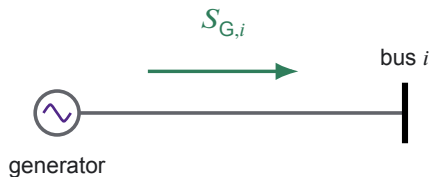
T2: bus 2 → bus 7

Line 7-5 → load at bus 5

One line represents three phases.

Source: Glover et al., 6th ed., Fig. 12.10.

Generator and load powers



$$S_{G,i} = P_{G,i} + jQ_{G,i}$$

Positive: power supplied to the bus.

$$S_{L,i} = P_{L,i} + jQ_{L,i}$$

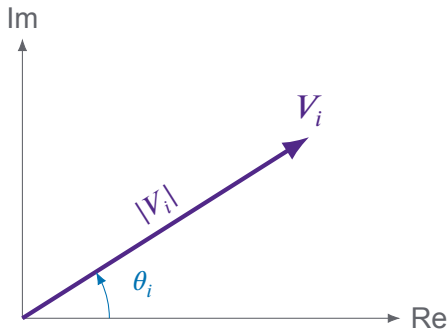
Positive: power withdrawn from the bus.

P : real power. Q : reactive power; its sign distinguishes supply from absorption.

For the typical study here, load $P_{L,i}$, $Q_{L,i}$ remain constant as voltage changes.

P : MW; Q : Mvar; $|S|$: MVA. Use the common three-phase base S_{base} .

Voltage at a bus



$$V_i = |V_i|e^{j\theta_i}$$

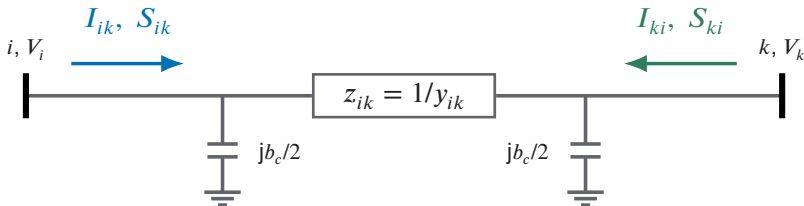
$|V_i|$: RMS voltage magnitude.

θ_i : angle relative to the common phase reference.

All terminals connected to the same bus share its voltage.

Transmission line quantities

A line has a current and a complex power at each terminal.



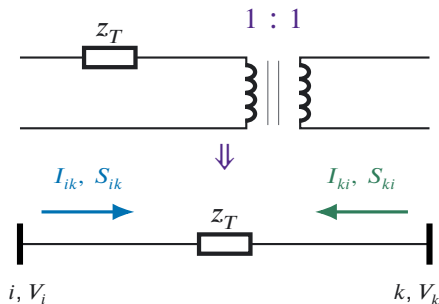
Both reference arrows point into the line. $S_{ik} = P_{ik} + jQ_{ik}$; similarly at bus k .

I_{ik}, I_{ki} are terminal currents, including the charging current at each end.

Model parameters: $z_{ik} = r_{ik} + jx_{ik}$, $y_{ik} = 1/z_{ik}$, and total charging b_c .

Source: Glover et al., 6th ed., Ch. 5.

Transformer quantities



Matched voltage bases

The nominal ideal ratio is $t = 1$ in per unit.

Excitation neglected

The winding resistance and leakage reactance remain:

$$z_T = r_T + jx_T$$

Current and power are referenced into the transformer at each terminal.

Retain off-nominal taps and phase shifts when present.

Source: Glover et al., 6th ed., Ch. 3.

Quantities in the network model

Object	Quantities
Bus i	V_i : magnitude $ V_i $ and angle θ_i
Generator	$S_{G,i} = P_{G,i} + jQ_{G,i}$
Load	$S_{L,i} = P_{L,i} + jQ_{L,i}$
Branch currents	I_{ik}, I_{ki} at the two terminals
Branch powers	$S_{ik} = P_{ik} + jQ_{ik}, S_{ki} = P_{ki} + jQ_{ki}$

Model parameters

Lines: z_{ik}, y_{ik}, b_c . Transformers: z_T , tap ratio t , and phase shift.

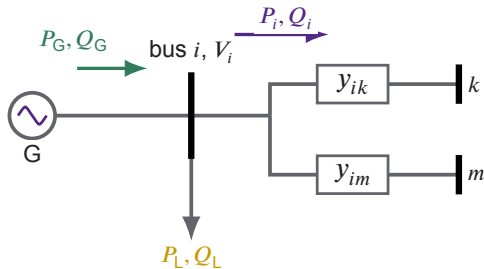
Next, use these quantities to define net injection and write the physical constraints.

PART 2

Physical equations

Connect the defined quantities through branch voltage and current relations, complex power, and current balance at each bus.

Net bus injection



Combine the device powers

$$P_i = P_{G,i} - P_{L,i}$$

$$Q_i = Q_{G,i} - Q_{L,i}$$

Positive: supplies the network.

Negative: withdraws power.

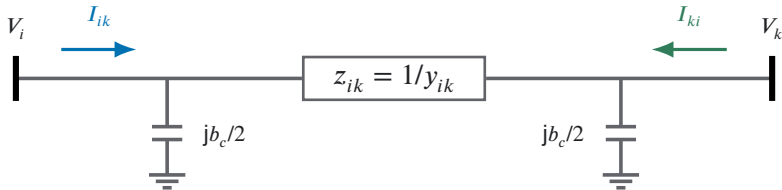
$$S_i = P_i + jQ_i = S_{G,i} - S_{L,i} = V_i I_i^*$$

I_i : net current entering the passive network; *: complex conjugate.

Use per-unit powers and voltages on consistent bases.

Branch current from endpoint voltages

Kirchhoff's voltage law (KVL) gives the series voltage $V_i - V_k$.



Ohm's law gives the series current; add the charging current at each end:

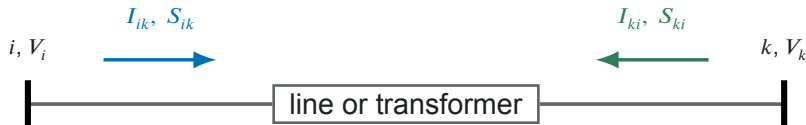
$$I_{ik} = y_{ik}(V_i - V_k) + j\frac{b_c}{2}V_i$$

$$I_{ki} = y_{ik}(V_k - V_i) + j\frac{b_c}{2}V_k$$

For the transformer equivalent with $t = 1$, use $y_{ik} = 1/z_T$ and omit charging.

Branch powers and losses

Complex power follows from the terminal voltage and current: $S = VI^*$.



$$S_{ik} = V_i I_{ik}^* = P_{ik} + jQ_{ik}$$

$$S_{ki} = V_k I_{ki}^* = P_{ki} + jQ_{ki}$$

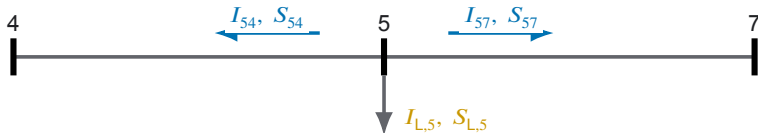
For transfer $i \rightarrow k$: sent power is S_{ik} ; delivered power is $-S_{ki}$.

$$P_{\text{loss},ik} = P_{ik} + P_{ki}$$

$Q_{ik} + Q_{ki}$ is net reactive absorption; line charging can make it negative.

From current balance to power balance

Apply Kirchhoff's current law (KCL) at bus 5; no separate bus shunt is present.



$$I_5 = -I_{L,5} = I_{54} + I_{57}$$

Conjugate the current equation and multiply by the common bus voltage V_5 :

$$S_5 = S_{54} + S_{57} = -S_{L,5}$$

The reference arrows point outward; a negative P_{5k} means power arrives at bus 5.

The equations connect devices to the network

1. Sum the currents leaving each bus

$$I_i(\mathbf{V}) = \sum_{k \in \mathcal{N}_i} I_{ik}(V_i, V_k) + y_{\text{sh},i} V_i$$

$\mathcal{N}_i$: connected neighbors; $y_{\text{sh},i}$: any separate bus shunt admittance.

2. Match device injection to network power

$$(P_{G,i} - P_{L,i}) + j(Q_{G,i} - Q_{L,i}) = V_i [I_i(\mathbf{V})]^*$$

$\mathbf{V}$ collects all bus voltages. Branch currents already include line charging.

The real and imaginary parts give two coupled power balance equations per bus.

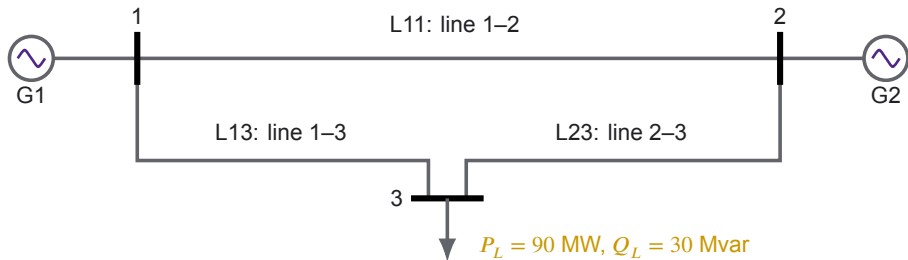
PART 3

Write the physical equations

Use the three-bus network to write branch currents and power balance at every bus. Keep the voltages symbolic; operating controls will be added later.

Exercise: given network

Balanced positive-sequence study on a 100-MVA base



Use the same three-bus network throughout the exercise.

Exercise: branch data

All branches contain only series impedance; no charging, separate shunts, or transformers.

Branch	z_{ik} (p.u.)	Series admittance
1–2	$0.018 + j0.060$	$y_{12} = 1/z_{12}$
1–3	$0.070 + j0.210$	$y_{13} = 1/z_{13}$
2–3	$0.045 + j0.135$	$y_{23} = 1/z_{23}$

Keep y_{12} , y_{13} , y_{23} symbolic when writing the equations.

Exercise: device powers

The 100-MVA base gives load demand $P_{L,3} = 0.90$, $Q_{L,3} = 0.30$ p.u.

Bus	Connected device	Net complex injection
1	Generator only	$S_1 = P_{G,1} + jQ_{G,1}$
2	Generator only	$S_2 = P_{G,2} + jQ_{G,2}$
3	Load; no generation	$S_3 = -0.90 - j0.30$

Keep V_1, V_2, V_3 and the generator powers symbolic.

Generator controls will be assigned later.

Exercise: write the equations

- 1 Write the current at each end of all three branches.
- 2 Apply KCL to express I_1, I_2, I_3 in terms of V_1, V_2, V_3 .
- 3 Use $S_i = V_i I_i^*$ to write complex power balance at every bus.
- 4 Write the complex power at both ends of each branch.

Submit the physical relations. No voltage solution or numerical calculation is required.

Reference: branch and bus currents

For each series branch $(i, k) \in \{(1, 2), (1, 3), (2, 3)\}$:

$$I_{ik} = y_{ik}(V_i - V_k), \quad I_{ki} = y_{ik}(V_k - V_i)$$

KCL at the three buses

$$I_1 = I_{12} + I_{13} = y_{12}(V_1 - V_2) + y_{13}(V_1 - V_3),$$

$$I_2 = I_{21} + I_{23} = y_{12}(V_2 - V_1) + y_{23}(V_2 - V_3),$$

$$I_3 = I_{31} + I_{32} = y_{13}(V_3 - V_1) + y_{23}(V_3 - V_2).$$

Each sum contains exactly the branches connected to that bus.

Reference: power balance at every bus

Match the connected device power to the network power at each bus:

$$P_{G,1} + jQ_{G,1} = V_1 [y_{12}(V_1 - V_2) + y_{13}(V_1 - V_3)]^* ,$$

$$P_{G,2} + jQ_{G,2} = V_2 [y_{12}(V_2 - V_1) + y_{23}(V_2 - V_3)]^* ,$$

$$-0.90 - j0.30 = V_3 [y_{13}(V_3 - V_1) + y_{23}(V_3 - V_2)]^* .$$

Each complex equality contains a real and a reactive power balance.

The physical relations hold before any voltage targets or generator schedules are chosen.

Reference: branch powers

For each branch $(i, k) \in \{(1, 2), (1, 3), (2, 3)\}$:

$$S_{ik} = V_i [y_{ik}(V_i - V_k)]^*$$

$$S_{ki} = V_k [y_{ik}(V_k - V_i)]^*$$

Real and reactive components are $P_{ik} = \text{Re}\{S_{ik}\}$ and $Q_{ik} = \text{Im}\{S_{ik}\}$.

The reference direction at each terminal points into the branch.

Bus power balance sums the terminal powers of the branches connected to that bus.

From a network to physical equations

1 Identify the devices and quantities

Record branch endpoints and parameters, device powers, and bus voltages.

2 Write branch and bus currents

Apply the branch models and sum connected terminal currents using KCL.

3 Write the power relations

Use $S = VI^*$ at branch terminals and buses. Match each net bus injection to generation minus load.

Next, collect the voltage coefficients into the bus admittance matrix.

Summary

- 1 Devices and buses carry voltage, current, and power quantities.
- 2 Branch models relate terminal currents to endpoint voltages.
- 3 KCL and complex power give the physical balance equations.

The same relations apply throughout the network. Operating specifications will later determine which quantities are given and which must be solved.

Build the network relation $\mathbf{I} = \mathbf{Y}_{\text{bus}} \mathbf{V}$ by collecting the current equations.